Appendix:

Mathematical models of the knee

Introduction

Mathematical models make it possible to calculate the values of quantities which are difficult or impossible to measure and provide insights which are not obtained from experiment alone. They are a necessary adjunct to the experimental method, but are not a common feature of biological or clinical research. A model is based on a series of assumptions or hypotheses about the way a physical system works. It is validated by comparing its predictions with independent experimental measurement. Reasonable validation then gives confidence in the assumptions on which the model is based and in the predictions of quantities which cannot be measured. The purpose in presenting our models here is to explain the differences between unloaded and loaded motion described in Chapter 3.

Many mathematical models of the natural knee [1–3] and of knee replacement have been proposed [4–6]. They have usually modelled the movement of the joint under load. We have found it easier to model the mobility of the knee in the absence of load, then the stability of the knee under load but in the absence of movement, and then to combine both to study activity, i.e. movement under load.

Three-dimensional model of knee mobility

The model (Fig. A1) explains the coupling of axial rotation to flexion and, more generally, how the joint achieves a range of unresisted passive mobility [7]. The model was formulated using the assumptions that the articular surfaces medially and laterally remain continuously in contact without interpenetration during passive flexion and that single fibres within each of the two cruciates and the MCL remain isometric. These five constraints to motion, when acting together, reduce the six possible degrees of freedom of the bones to one, so that axial rotation is coupled to flexion. The calculated coupling of axial rotation to flexion required to satisfy these assumptions agrees well with measurements described in Chapter 3 (Fig. A2).

Pictures of the model in extension and at 45° and 90° flexion (Fig. A1) show that during flexion the articular surfaces of the femur roll and slide on the surfaces of the tibia while the femur rotates externally on the tibia. The three isometric ligament fibres rotate about their insertions into the tibia. The sliding and rolling of the surfaces and the rotation of the ligament fibres are accomplished without tissue deformation and therefore without generating resistance to motion.

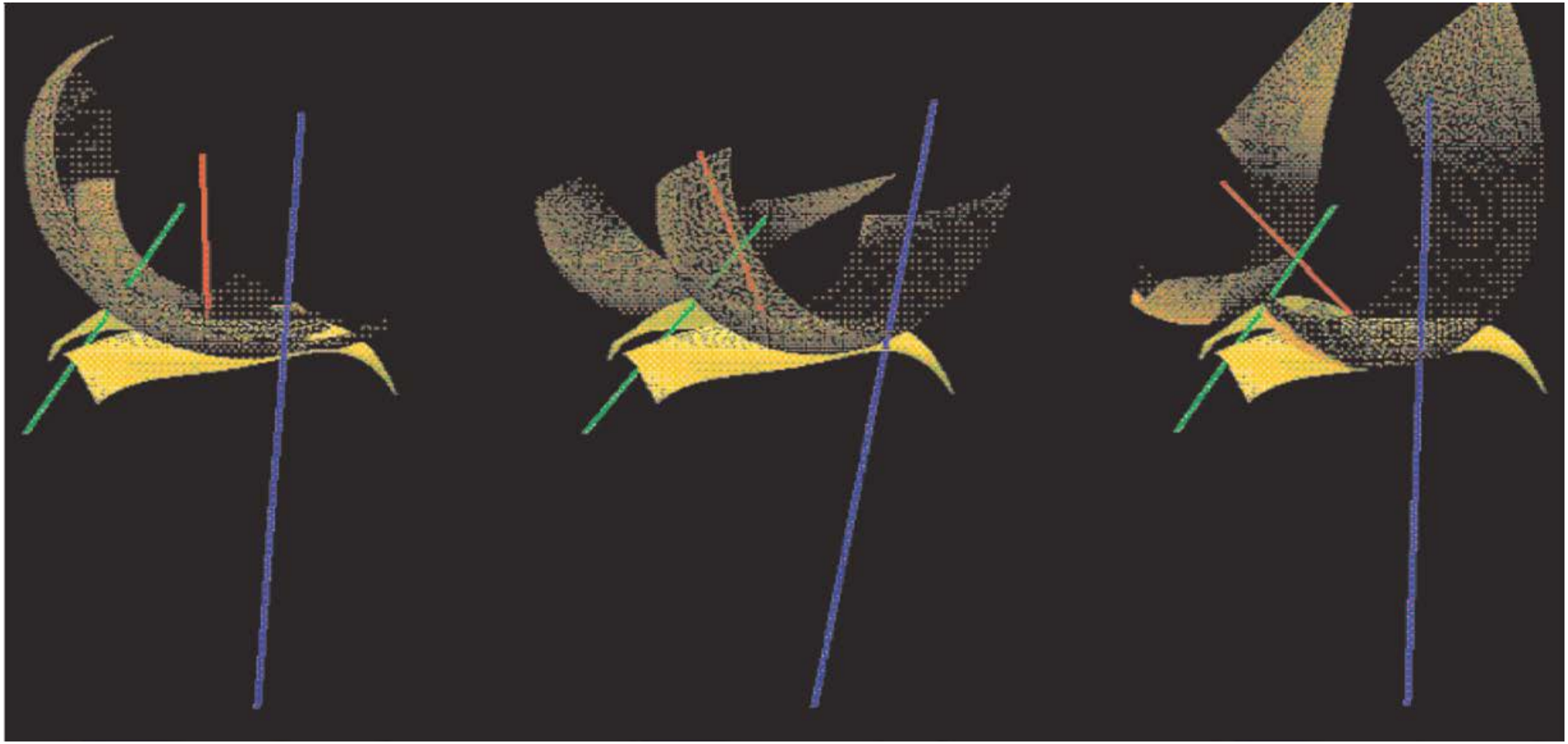

Figure A1 Three-dimensional model of knee mobility at extension, 45° flexion, and 90° flexion. The brown shells are the surfaces of the model femoral condyles, the yellow shells are the surfaces of the tibial plateaux, and the red, green and blue lines are the isometric fibres of the ACL, PCL, and MCL, respectively.

An animation of the model (Animation 1) can be found on the website accompanying this book, www.oxfordpartialknee.com, and demonstrates how the surfaces of the bones move on each other. The axial rotation of the surfaces of the femoral condyles can best be appreciated by watching the movements of their posterior edges.

The predictions of the model are sensitive to the choice of its parameters: (1) the shapes of the articular surfaces and (2) the positions of the points of origin and insertion of the isometric ligament fibres. If spherical femoral condyles and flat tibial surfaces are chosen, the result is greater anteroposterior movement of the contact points on the tibia in both compartments and larger values of coupled external femoral rotation. The model also demonstrates the important role of the MCL, and the absence of a role for the LCL, in guiding passive motion. If the model MCL is placed on the lateral rather than the medial side, obligatory internal rather than external rotation of the femur occurs during flexion, a result that is unlikely to be deduced except by modelling.

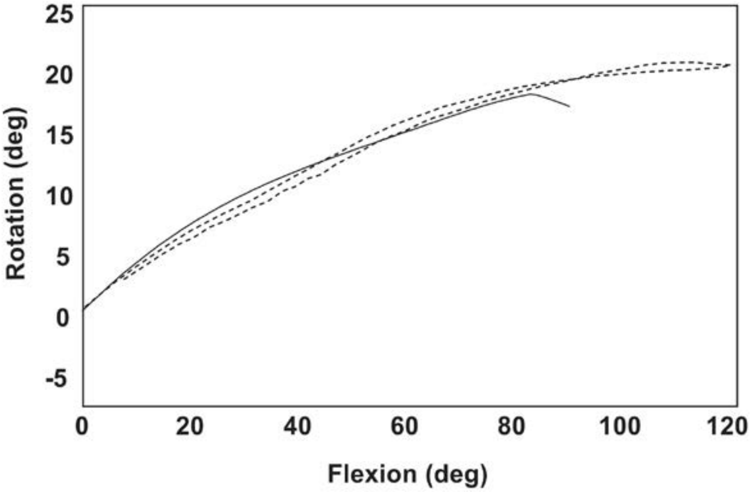

Figure A2 External rotation of the femur plotted against flexion angle. The solid line calculated from the three- dimensional model of mobility (Fig. A1) matches well the experimental curves of Figs. 3.2 and 3.3 (dotted lines).

Two-dimensional model of the knee: the four-bar linkage

Mobility

The rolling and sliding movements of the femur on the tibia are more clearly seen in the simpler two-dimensional four-bar linkage model of Figure A3 (website, Animation 2) [8–11]. The figure shows the femur flexing on a fixed tibia with lines representing the extensor and flexor muscles. The distal surface of the femur has separate curves representing (1) the sulcus of the trochlea, anteriorly, in contact with the patella, and (2) the distal and posterior facets of the condyle in contact